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Exclusive disjunctive soft sets

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ABSTRACT

Soft sets theory, initiated by Molodtsov, is an emerging tool to deal with uncertain problems and has been studied by scholars in both theory and practice. This paper proposes the notion of exclusive disjunctive soft sets and studies some of its operations, such as, restricted/relaxed *AND* operations, dependency between exclusive disjunctive soft sets and bijective soft sets, exclusive disjunctive soft decision systems, reduction of exclusive disjunctive soft decision systems, core of exclusive disjunctive soft decision systems, decision rules in exclusive disjunctive decision soft sets. Moreover, this study gives an application of exclusive disjunctive soft sets, which shows that it can be applied to attribute reduction of incomplete information system.

1. Introduction

Recently, soft sets theory has been widely focused by many scholars in theory and practice, which was initiated by Molodtsov [1]. In addition, it is an emerging tool to deal with uncertain problems. Based on the work of Molodtsov, Maji and Biswas [2] defined equality of two soft sets, subset and superset of soft set, complement of a soft set, null soft set, and absolute soft set with examples. They also defined soft binary operation such as *AND*, *OR* and the operation of Union, Intersection and De Morgan's law. Aktaş and Çağman [3] introduced the basic properties of soft sets to the related concept of fuzzy sets as well as rough sets, and then they gave a definition of soft group and derived basic properties by using Molodtsov's definition of the soft sets. Chen and Tsang [4] studied the parameterization reduction of soft sets and its application. Xiao and Chen [5] studied recognition for soft information based on the theory of soft sets. Liu and Yan [6] discussed the algebraic structure of fuzzy soft sets and gave the definition of fuzzy soft group. Jun and Park [7] proposed the notion of soft ideals and idealistic soft BCK/BCI-algebras, and gave several examples. They also provided the relations between soft BCK/BCI-algebras and idealistic soft BCK/BCI-algebras and established Intersection, Union, *AND* operation, and *OR* operation of soft ideals and idealistic soft BCK/BCI-algebras. Feng and Jun [8] introduced the notions of soft semirings, soft subsemirings, soft ideals, idealistic soft semirings and soft semiring homomorphisms. Ali and Feng et al. [9] corrected some errors of former studies and proposed some new operations on soft sets. Yang and Lin et al. [10] introduced the concept of the interval-valued fuzzy soft set by combining the interval-valued fuzzy set and soft set models. Aygünoglu and Aygün [11] introduced the concept of fuzzy soft group. The practice of soft sets theory was also extended to data analysis under incomplete information [12], combined forecasts [13], decision-making problems [14], and normal parameter reduction [15], and d-algebras [16]. As for the relationship between soft sets theory and rough sets [17–20], some effort has been done to such issues by scholars. Pei and Miao [21] represented rough set model as two soft sets. Aktaş and çağman [3] have proved that every rough set may be considered a soft set. Herawan and Deris [22] devoted to revealing interconnection between rough sets and soft sets

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Table 2.1
The tabular representation of (F, E).

U	e_1	e_2	e_3	e_4
h_1	1	1	1	0
h_2	0	1	0	1
h_3	1	0	1	1
h_4	1	0	0	1

and presented a direct proof that Pawlak's and Iwinski's rough sets can be considered as soft sets. Feng and Li et al. [23,24] initiated concepts of soft–rough fuzzy sets, rough–soft sets, soft–rough sets and soft–rough fuzzy sets.

Xiao and Gong et al. [25] proposed the notion of bijective soft set and implement some operations of classical rough sets [17,20] by means of bijective soft sets. Based on their research, this paper initiates the concept of exclusive disjunctive soft sets, which is an extended concept of bijective soft set, and this paper also studies some operations of it, such as, restricted/relaxed *AND* operations on an exclusive disjunctive soft set and a subset of universe, dependency between exclusive disjunctive soft set and bijective soft set, exclusive disjunctive soft decision system, reduction of exclusive disjunctive soft decision system, core of exclusive disjunctive soft decision system, decision rules. Moreover, this paper gives an application of exclusive disjunctive soft sets, which shows that it can be applied to attribute reduction of incomplete information system.

The rest of the paper is organized as follows. Section 2 introduces the basic principles of soft sets and bijective soft sets. Section 3 proposes the concepts of exclusive disjunctive soft sets, and some operations of it. Section 4 gives an application of exclusive disjunctive soft sets. Finally Section 5 presents some conclusions from this study.

2. Preliminaries

2.1. Soft sets

Let U be an initial universe and let E be a set of parameters.

Definition 2.1 (*[1]*). A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U, where F is a mapping given by $F : E \to P(U)$.

In other words, the soft set is a parameterized family of subsets of the set U. Every set $F(\varepsilon)(\varepsilon \in E)$, from this family may be considered the set of ε-elements of the soft sets (F, E), or as the set of ε-approximate elements of the soft set.

To illustrate this idea, let us consider the following example.

Example 1. Let universe $U = \{h_1, h_2, h_3, h_4\}$ be a set of houses, a set of parameters $E = \{e_1, e_2, e_3, e_4\}$ be a set of status of houses which stand for the parameters "beautiful", "cheap", "in green surroundings", and "in good location" respectively. Consider the mapping F be a mapping of E into the set of all subsets of the set U. Now consider a soft set (F, E) that describes the "attractiveness of houses for purchase". According to the data collected, the soft set (F, E) is given by

$$(F, E) = \{(e_1, \{h_1, h_3, h_4\}), (e_2, \{h_1, h_2\}), (e_3, \{h_1, h_3\}), (e_4, \{h_2, h_3, h_4\})\},$$

where $F(e_1) = \{h_1, h_3, h_4\}$, $F(e_2) = \{h_1, h_2\}$, $F(e_3) = \{h_1, h_3\}$ and $F(e_4) = \{h_2, h_3, h_4\}$. In order to store a soft set in computer, a two-dimensional table is used to represent the soft set (F, E). Table 2.1 is the tabular form of the soft set (F, E). If $h_i \in F(e_j)$, then $h_{ij} = 1$, otherwise $h_{ij} = 0$, where h_{ij} are the entries (see Table 2.1).

Definition 2.2 (*See [2]*). For two soft sets (F, A) and (G, B) over U, (F, A) is called a soft subset of (G, B) if

(1) $A \subset B$ and
(2) $\forall \varepsilon \in A$, $F(\varepsilon)$ and $G(\varepsilon)$ are identical approximations.

This relationship is denoted by $(F, A) \tilde{\subset} (G, B)$.

Similarly, (F, A) is called a soft superset of (G, B) if (G, B) is a soft subset of (F, A). This relationship is denoted by $(F, A) \tilde{\supset} (G, B)$.

Definition 2.3 (*See [2]*). Two soft sets (F, A) and (G, B) over U are called soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 2.4 (*See [2]*). The intersection of two soft sets (F, A) and (G, B) over U is the soft set (H, C), where $C = A \cap B$ and $\forall \varepsilon \in C, H(\varepsilon) = F(\varepsilon)$ or $G(\varepsilon)$ (as both are same set). This is denoted by $(F, A) \tilde{\cap} (G, B) = (H, C)$.

Definition 2.5 (*See [2]*). The union of two soft sets (F, A) and (G, B) over U is the soft set (H, C), where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B. \end{cases} \tag{2.1}$$

This is denoted by $(F, A) \tilde{\cup} (G, B) = (H, C)$.

Definition 2.6 (*See [2]*). **AND** operation on Two Soft Sets. If (F, A) and (G, B) are two soft sets then "(F, A) AND (G, B)" denoted by $(F, A) \wedge (G, B)$ is defined by $(F, A) \wedge (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cap G(\beta), \forall(\alpha, \beta) \in A \times B$.

Definition 2.7 (*See [2]*). NULL SOFT SET. A soft set (F, A) over U is said to be a NULL soft set denoted by Φ, if $\varepsilon \in A$, $F(\varepsilon) = \emptyset$.

2.2. Bijective soft sets

Bijective soft sets [25] is a subtype of soft sets with some good characteristics. It can help to transform a single-valued information system into soft set. To illustrate the concept of bijective soft sets, we will give an example below firstly. And it will be used to illustrate some notions of it.

Example 2. Let $U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ be a common universe, which is a set of six stores. Suppose that the six stores are characterized by a soft set (F, E) over a common universe U. E denotes the parameter set, $E = E_1 \cup E_2 \cup E_3 \cup E_4$. E_1 describes the empowerment of sales personnel. E_2 describes the perceived quality of merchandise. E_3 describes the high traffic location. And E_4 describes store profit or loss. The parameter sets of these parameter sets are $E_1 = \{\text{high, med., low}\}$, $E_2 = \{\text{good, avg.}\}$, $E_3 = \{\text{no, yes}\}$ and $E_4 = \{\mathit{profit}, \mathit{loss}\}$, respectively. And (F_i, E_i) is soft subset of (F, E), where $i = 1, 2, 3, 4$.

The mapping of each soft sets over U defined as follows:

$$F_1(\text{high}) = \{x_1, x_6\}, \quad F_1(\text{med.}) = \{x_2, x_3, x_5\}, \quad F_1(\text{low}) = \{x_4\}$$
$$F_2(\text{good}) = \{x_1, x_2, x_3\}, \quad F_2(\text{avg.}) = \{x_4, x_5, x_6\}$$
$$F_3(\text{no}) = \{x_1, x_2, x_3, x_4\}, \quad F_3(\text{yes}) = \{x_6, x_5\}$$
$$F_4(\text{profit}) = \{x_1, x_3, x_6\}, \quad F_4(\text{loss}) = \{x_2, x_4, x_5\}.$$

Definition 2.8 (*[25]*). Let (F, B) be a soft set over a common universe U, where F is a mapping $F : B \to P(U)$ and B is nonempty parameter set. We say that (F, B) is a bijective soft set, if (F, B) such that

(i) $\bigcup_{e \in B} F(e) = U$.
(ii) For any two parameters $e_i, e_j \in B$, $e_i \neq e_j$, $F(e_i) \cap F(e_j) = \emptyset$.

In other words, suppose $Y \subseteq P(U)$ and $Y = \{F(e_1), F(e_2), \dots, F(e_n)\}$, $e_1, e_2, \dots, e_n \in B$. From Definition 2.8, the mapping $F : B \to P(U)$ can be transformed to the mapping $F : B \to Y$, which is a bijective function. i.e. for every $y \in Y$, there is exactly one parameter e in B such that $F(e) = y$ and no unmapped element remains in both B and Y. Simply, every element in bijective soft set over U can have one and only one mapping to a parameter.

Example 3. Suppose that $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$ is a common universe, (F, E) is a soft set over U, $E = \{e_1, e_2, e_3, e_4\}$. The mapping of (F, E) is given below:

$$F(e_1) = \{x_1, x_2, x_3\}$$
$$F(e_2) = \{x_4, x_5, x_6\}$$
$$F(e_3) = \{x_7\}$$
$$F(e_4) = \{x_4, x_5, x_6, x_7\}.$$

From Definition 2.8, $(F, \{e_1, e_2, e_3\})$ and $(F, \{e_1, e_4\})$ are bijective soft sets. While $(F, \{e_1, e_2\})$ and $(F, \{e_1, e_3\})$ are not bijective soft sets.

Theorem 2.1 (*[25]*). *Suppose that (F, E) and (G, B) are two bijective soft sets over common universe U. $(H, C) = (F, E) \wedge (G, B)$ is a bijective soft set.*

Theorem 2.2 (*[25]*). *Suppose that (F, E) is a bijective soft set over U and (G, B) is a Null soft set over U. $(H, C) = (F, E) \tilde{\cup} (G, B)$ is a bijective soft set.*

3. Exclusive disjunctive soft sets

To formulate the concept of exclusive disjunctive soft sets, we will give an example below firstly. And it will be used to illustrate some notions of this section.

Example 4. Let $U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ be a common universe, which is a set of six stores. Suppose that the six stores are characterized by a soft sets (F, E) over a common universe U. E denotes the parameter set, $E = E_1 \cup E_2 \cup E_3 \cup E_4$ and any parameter of these are disjoint. E_1 describes the empowerment of sales personnel. E_2 describes the perceived quality of merchandise. E_3 describes the high traffic location. E_4 describes store profit or loss. And the parameter sets of the these are $E_1 = \{\text{high, med., low}\}$, $E_2 = \{\text{good, avg.}\}$, $E_3 = \{\text{no, yes}\}$ and $E_4 = \{\text{profit, loss}\}$, respectively. And (F_i, E_i) is soft subset of (F, E), where $i = 1, 2, 3, 4$.

Table 3.1
The exclusive-disjunctively interpreted tabular form of (F_1, E_1).

	High	Med.	Low
x_1	ξ_{x_1}(high)	ξ_{x_1}(med.)	0
x_2	0	1	0
x_3	0	1	0
x_4	0	ξ_{x_4}(med.)	ξ_{x_4}(low)
x_5	0	1	0
x_6	1	0	0

Each soft sets over U have their mapping, which is defined as follows:

$$F_1(\text{high}) = \{x_1, x_6\}, \qquad F_1(\text{med.}) = \{x_1, x_2, x_3, x_4, x_5\}, \qquad F_1(\text{low}) = \{x_4\}$$
$$F_2(\text{good}) = \{x_1, x_2, x_3\}, \qquad F_2(\text{avg.}) = \{x_4, x_5, x_6\}$$
$$F_3(\text{no}) = \{x_1, x_2, x_3, x_4\}, \qquad F_3(\text{yes}) = \{x_4, x_5\}$$
$$F_4(\text{profit}) = \{x_1, x_2, x_3, x_6\}, \qquad F_4(\text{loss}) = \{x_2, x_4, x_5\}.$$

3.1. Concept of exclusive disjunctive soft sets

Definition 3.1. Let (F, E) be a soft set over a common universe U, x be an element of U and e be a parameter of E. $\xi_x(e)$ denotes the characteristic function of e-element of the soft set (F, E), defined by

$$\xi_x(e) = \begin{cases} 1 & \text{if } x \in F(e) \\ 0 & \text{if } x \notin F(e). \end{cases} \tag{3.1}$$

Definition 3.2. Let (F, B) be a soft set over a common universe U, where F is a mapping $F : B \to P(U)$ and B is nonempty parameter set. We say that (F, B) is an exclusive disjunctive soft set, if (F, B) such that

(i) $\bigcup_{e\in B} F(e) = U$.
(ii) Let x be an element of universe. If $\sum_{e\in B} \xi_x(e) > 1$ then $\xi_x(e)$ is exclusive-disjunctively interpreted as:
  (a) each $\xi_x(e)$ is unknown but $\sum_{e\in B} \xi_x(e) = 1$.
  (b) And x is called an uncertain element of (F, B).

In other words, when an element x exists more than one parameter mapping ($x \in F(e_i)$ and $x \in F(e_j) \dots$) the 'exclusive-disjunctively interpreted' means x can belong to $F(e_i)$ or $F(e_j)$ but x can only belong to one of them.

Obviously, a bijective soft set is a special exclusive disjunctive soft set, which do not contain uncertain element. To illustrate this notion vividly, we will give an example below.

Example 5. Let us reconsider the soft sets given in Example 4. From Definition 3.2, (F_1, E_1) is an exclusive disjunctive soft set. There exists $x_1 \in F_1(\text{high}), x_1 \in F_1(\text{med.})$ and $x_4 \in F_1(\text{med.}), x_4 \in F_1(\text{low})$. This can be exclusive-disjunctively interpreted as: x_1 can be with high empowerment of sales personnel or med. empowerment of sales personnel, but x_1 can only be with one of them. It can be represented as a tabular form in Table 3.1. Note that, since $x_1 \in F_1(\text{high}), x_1 \in F_1(\text{med.})$ is not certain we just use unknown characteristic function of them, such as $\xi_{x_1}(\text{high})$ and $\xi_{x_1}(\text{med.})$, to represented in the corresponding cells. Although we do not know the value of them we can confirm that $\xi_{x_1}(\text{high}) + \xi_{x_1}(\text{med.}) = 1$. This is an exclusive-disjunctively interpreted example of $x_1 \in F_1(\text{high}), F_1(\text{med.})$.

Definition 3.3. Suppose that (F, B) is an exclusive disjunctive soft set and $e \in B$ is a parameter of (F, B). $\tilde{F}(e) = \{x | x \in F(e)$ and x is an uncertain element of $(F, B)\}$ is called the uncertain element set of $F(e)$. And $\underset{\sim}{F}(e) = F(e) - \tilde{F}(e)$ is called the certain element set of $F(e)$.

Example 6. Let us reconsider the soft sets given in Example 4. From Definition 3.2, we can write $\tilde{F}(\text{high}) = \{x_1\}$ and $\underset{\sim}{F}(\text{high}) = \{x_6\}$.

Theorem 3.1. *Suppose that (F, E) and (G, B) are two exclusive disjunctive soft sets over common universe U. $(H, C) = (F, E) \wedge (G, B)$ is an exclusive disjunctive soft set.*

Proof. From Definition 2.6, $(F, A) \wedge (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cap G(\beta), \forall(\alpha, \beta) \in A \times B$. Suppose $e \in A \times B$ is a parameter of (H, C).

$$\because H(e) = F(\alpha) \cap G(\beta)$$
$$\therefore \bigcup_{e\in B} H(e) = \bigcup_{\alpha\in A} \bigcup_{\beta\in B} F(\alpha) \cap G(\beta) = \bigcup_{\alpha\in A} F(\alpha) \cap \left(\bigcup_{\beta\in B} G(\beta)\right) = \bigcup_{\alpha\in A} F(\alpha) \cap U = U.$$

For an uncertain element x, since $e = A \times B$ and $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$, $\forall(\alpha, \beta) \in A \times B$

$$\sum_{e \in C} \xi_x(e) = \sum_{a \in A} \sum_{b \in B} \xi_x(a)\xi_x(b) = 1.$$

Therefore, $(H, C) = (F, E) \wedge (G, B)$ is an exclusive disjunctive soft set. □

Theorem 3.2. *Suppose that (F, E) is an exclusive disjunctive soft set over U and (G, B) is a Null soft set over U. $(H, C) = (F, E) \tilde{\cup} (G, B)$ is an exclusive disjunctive soft set.*

Proof. Straightforward. □

3.2. Some operations on exclusive disjunctive soft sets

Definition 3.4 (*Restricted AND Operation on an Exclusive Disjunctive Soft Set and a Subset of Universe*). Let $U = \{x_1, x_2, \ldots, x_n\}$ be a common universe, X be a subset of U, and (F, E) be an exclusive disjunctive soft set over U. The operation of "(F, E) restricted AND X" is denoted by $(F, E) \underset{\sim}{\wedge} X$ and is given by

$$\bigcup_{e \in E} (\{\underset{\sim}{F}(e) | \underset{\sim}{F}(e) \in X\} \cup \{x | \underset{\sim}{F}(e) \in X, x \in \tilde{F}(e) \cap X\}). \tag{3.2}$$

Example 7. Let (G, B) be an exclusive disjunctive soft set over a common universe U, $U = \{x_1, x_2, x_3, x_4\}$. Suppose the following:

$$(G, B) = \{e_1 = \{x_1\}, e_2\{x_2, x_3/\xi_{x_3}(e_2), x_4/\xi_{x_4}(e_2)\}, e_3\{x_3/\xi_{x_3}(e_3), x_4/\xi_{x_4}(e_3)\}\}$$
$$X = \{x_2, x_3\}$$

where $x_3/\xi_{x_3}(e_2)$ denotes that x_3 has an unknown characteristic function of e_2 and is written as $\xi_{x_3}(e_2)$ (see Definition 3.1).

From Definition 3.4, we can write $(G, B) \underset{\sim}{\wedge} X = \{x_2, x_3/\xi_{x_3}(e_2), x_3/\xi_{x_3}(e_3)\}$. Since $\xi_{x_3}(e_2) + \xi_{x_3}(e_3) = 1$ we can write $(G, B) \underset{\sim}{\wedge} X = \{x_2, x_3\}$.

Definition 3.5 (*Relaxed AND Operation on a Soft Set and a Subset of Universe*). Let set $U = \{x_1, x_2, \ldots, x_n\}$ be a common universe, X be a subset of U, and (F, E) be an exclusive disjunctive soft set over U. The operation of "(F, E) relaxed AND X" is denoted by $(F, E) \tilde{\wedge} X$ and is given by

$$\bigcup_{e \in E} \{F(e) : F(e) \cap X \neq \emptyset\}. \tag{3.3}$$

Example 8. Let (G, B) be an exclusive disjunctive soft set over a common universe U, $U = \{x_1, x_2, x_3, x_4\}$. Suppose the following:

$$(G, B) = \{e_1 = \{x_1, x_2/\xi_{x_2}(e_1)\}, e_2\{x_2/\xi_{x_2}(e_2), x_3, x_4\}\}$$
$$X = \{x_2, x_3\}.$$

From Definition 3.5, we can write $(G, B) \tilde{\wedge} X = \{x_1, x_2/\xi_{x_2}(e_1), x_2/\xi_{x_2}(e_2), x_3, x_4\}$. Since $\xi_{x_2}(e_1) + \xi_{x_2}(e_2) = 1$ we can write $(G, B) \tilde{\wedge} X = \{x_1, x_2, x_3, x_4\}$.

The boundary region of the exclusive disjunctive soft set (G, B) with respect to X is

$$(G, B) \tilde{\wedge} X - (G, B) \underset{\sim}{\wedge} X = \{x_1, x_4\}. \tag{3.4}$$

3.3. Dependency between exclusive disjunctive soft sets and bijective soft sets

Definition 3.6 (*Dependency between Exclusive Disjunctive Soft Sets and Bijective Soft Sets*). Suppose that (F, E) is an exclusive disjunctive soft set over a common universe U and (D, C) is a bijective soft set over U, where $E \cup C = \emptyset$. (F, E) is said to depend on (D, C) to a degree k $(0 \leq k \leq 1)$, denoted by $(F, E) \underset{\sim}{\Rightarrow}_k (D, C)$, if

$$k = \gamma((F, E), (D, C)) = \frac{\sum_{x \in P, e \in E} \xi_x(e)}{|U|} \tag{3.5}$$

where $P = \bigcup_{e \in C} (F, E) \underset{\sim}{\wedge} D(e)$.

If $k = 1$ we say (F, E) is full depended on (D, C).
If $k = 0$ we say (F, E) is not depended on (D, C).

Note that, since the uncertain characteristic function may exist in the result of restricted *AND* operation, the value of k may be a set of enumerative values.

Table 3.2
The tabular form of $(F_1, E_1) \wedge (F_2, E_2) \wedge (F_3, E_3)$.

	e_1	e_2	e_3	e_4	e_5	e_6
x_1	1	1	0	0	0	0
x_2	0	1	0	0	0	0
x_3	0	1	0	0	0	0
x_4	0	0	1	0	0	1
x_5	0	0	0	1	0	0
x_6	0	0	0	0	1	0

Note that, those parameters which have no element to be mapped are not included in this table.
Where e_1 = high and good and no; e_2 = med and good and no; e_3 = low and avg. and no; e_4 = med and avg. and yes; e_5 = high and avg. and yes; e_6 = med. and avg. and no.

To illustrate this concept, we will give an example below.

Example 9. Let us reconsider the exclusive disjunctive soft sets given in Example 4.

$$F_4(\text{profit}) = \{x_1, x_2, x_3, x_6\} \qquad F_4(\text{loss}) = \{x_4, x_5\}$$
$$(F_1, E_1) = \{\{x_1/\xi_{x_1}(\text{high}), x_6\}, \{x_1/\xi_{x_1}(\text{med.}), x_2, x_3, x_4/\xi_{x_4}(\text{med.}), x_5\}, \{x_4/\xi_{x_4}(\text{low})\}\}$$
$$(F_1, E_1) \underset{\sim}{\wedge} F_4(\text{profit}) = \{x_1/\xi_{x_1}(\text{high}), x_6\}$$
$$(F_1, E_1) \underset{\sim}{\wedge} F_4(\text{loss}) = \{x_4/\xi_{x_4}(\text{low})\}$$

where $x_1/\xi_{x_1}(\text{high})$ denotes that x_1 has an unknown characteristic function of parameter 'high' and is written as $\xi_{x_1}(\text{high})$ (see Definition 3.1).

From Definition 3.6, we can write

$$k = \gamma((F_1, E_1), (F_4, D_4)) = \frac{1 + \xi_{x_1}(\text{high}) + \xi_{x_4}(\text{low})}{|U|} = \frac{1 + \chi^{(2)}}{|U|}.$$

Since $\xi_{x_1}(\text{high})$ and $\xi_{x_4}(\text{low})$ is 0 or 1, $k = \gamma((F_1, E_1), (F_4, D_4)) = \{\frac{1}{2}, \frac{1}{3}, \frac{1}{6}\}$.

Note that, $\chi^{(n)}$ denotes that there are n unknown characteristic functions in dependency formula.

3.4. Concept of exclusive disjunctive soft decision system

Definition 3.7 (*Exclusive Disjunctive Soft Decision System*). Suppose that (F_i, E_i) $(i = 1, 2, 3, \ldots, n)$ are n exclusive disjunctive soft sets over a common universe U, where any $E_i \cap E_j = \emptyset$ $(i = 1, 2, 3, \ldots, n; j = 1, 2, 3, \ldots n; i \neq j)$, (G, B) is a bijective soft set over a common universe U, $B \cap E_i = \emptyset$ $(i = 1, 2, 3, \ldots, n)$, and we call it the decision soft set. Suppose $(F, E) = \tilde{\bigcup}_{i=1}^{n}(F_i, E_i)$.The triple $((F, E), (G, B), U)$ is called exclusive disjunctive soft decision system over a common universe U.

In Example 4, we consider an exclusive disjunctive soft decision system $(\tilde{\bigcup}_{i=1}^{3}(F_i, E_i), (F_4, E_4), U)$. This exclusive disjunctive soft decision system set describes the profit ability of stores and other information.

Definition 3.8 (*Exclusive Disjunctive Soft Decision System Dependency*). Let $((F, E), (G, B), U)$ be an exclusive disjunctive soft decision system, where $(F, E) = \tilde{\bigcup}_{i=1}^{n}(F_i, E_i)$ and (F_i, E_i) is an exclusive disjunctive soft set. (F, E) is called condition soft set. The soft dependency between $(F_1, E_1) \wedge (F_2, E_2) \wedge \cdots \wedge (F_n, E_n)$ and (G, B) is called soft decision system dependency of $((F, E), (G, B), U)$, denoted by κ and given by

$$\kappa = \gamma\left(\bigwedge_{i=1}^{n}(F_i, E_i), (G, B)\right). \tag{3.6}$$

Example 10. Let us reconsider the exclusive disjunctive soft sets given in Example 4. Suppose that $(\tilde{\bigcup}_{i=1}^{n}(F_i, E_i), (F_4, E_4), U)$ is an exclusive disjunctive soft decision system on how to choose profitable stores. The tabular form of $(F_1, E_1) \wedge (F_2, E_2) \wedge (F_3, E_3)$ is given in Table 3.2.

Suppose that $(F_1, E_1) \wedge (F_2, E_2) \wedge (F_3, E_3) = (H, C)$

$$F_4(\text{profit}) = \{x_1, x_2, x_3, x_6\} \qquad F_4(\text{loss}) = \{x_4, x_5\}$$
$$(H, C) = \{\{x_1/\xi_{x_1}(e_1)\}, \{x_1/\xi_{x_1}(e_2), x_2, x_3\}, \{x_4/\xi_{x_4}(e_3)\}, \{x_5\}, \{x_6\}, \{x_4/\xi_{x_4}(e_6)\}\}$$
$$(F_1, E_1) \underset{\sim}{\wedge} F_4(\text{profit}) = \{x_1/\xi_{x_1}(e_1), x_1/\xi_{x_1}(e_2), x_2, x_3, x_6\} = \{x_1, x_2, x_3, x_6\}$$
$$(F_1, E_1) \underset{\sim}{\wedge} F_4(\text{loss}) = \{x_4/\xi_{x_4}(e_3), x_4/\xi_{x_4}(e_6), x_5\} = \{x_4, x_5\}.$$

From Definition 3.8, we can write

$$\kappa = \gamma((H, C), (F_4, D_4)) = \frac{1 + 1 + 1 + 1 + \xi_{x_1}(e_1) + \xi_{x_1}(e_2) + \xi_{x_4}(e_3) + \xi_{x_4}(e_6)}{|U|} = 1.$$

Note that $\xi_{x_1}(e_1) + \xi_{x_1}(e_2) = 1$ and $\xi_{x_4}(e_3) + \xi_{x_4}(e_6) = 1$.

3.5. Reduction of exclusive disjunctive soft decision system

Definition 3.9. Let $((F, E), (G, B), U)$ be an exclusive disjunctive soft decision system, where $(F, E) = \tilde{\bigcup}_{i=1}^{n}(F_i, E_i)$ and (F_i, E_i) is an exclusive disjunctive soft set, $\bigcup_{i=1}^{m}(F_i, E_i) \tilde{\subset} (F, E)$. κ is the soft decision system dependency of $((F, E), (G, B), U)$. If $\max(\gamma(\wedge_{i=1}^{m}(F_i, E_i), (G, B))) = \kappa$ we say $\bigcup_{i=1}^{m}(F_i, E_i)$ is a reduct of exclusive disjunctive soft decision system $((F, E), (G, B), U)$.

Example 11. Let us reconsider the exclusive disjunctive soft set given in Example 4. Suppose that $(\tilde{\cup}_{i=1}^{3}(F_i, E_i), (F_4, E_4), U)$ is an exclusive disjunctive soft decision system on how to choose profitable stores. $\gamma((F_1, E_1) \wedge (F_2, E_2), (F_4, E_4)) = \{1, 9/10\}$ and $\max(\gamma((F_1, E_1) \wedge (F_2, E_2), (F_4, E_4))) = \kappa = 1$. Thus $(F_1, E_1) \tilde{\cup} (F_2, E_2)$ is a reduct of $(\tilde{\cup}_{i=1}^{3}(F_i, E_i), (F_4, E_4), U)$.

3.6. Core exclusive disjunctive soft set of exclusive disjunctive decision soft set

Definition 3.10. suppose that the exclusive disjunctive soft set (H, C) belong to every reduct of exclusive disjunctive soft decision system $((F, E), (G, B), U)$. We say (H, C) is a core exclusive disjunctive soft set of $((F, E), (G, B), U)$.

3.7. Decision rules in exclusive disjunctive decision system

Definition 3.11. Let $((F, E), (G, B), U)$ be an exclusive disjunctive soft decision system, where $(F, E) = \tilde{\bigcup}_{i=1}^{n}(F_i, E_i)$ and (F_i, E_i) is an exclusive disjunctive soft set, $\bigcup_{i=1}^{m}(F_i, E_i) \tilde{\subset} (F, E)$ is a reduct of exclusive disjunctive soft decision system $((F, E), (G, B), U)$. Suppose that $(H, C) = \wedge_{i=1}^{m}(F_i, E_i)$. We call

$$\text{if } e_i \text{ then } e_j(|H(e_i)|/|G(e_j)|). \tag{3.7}$$

A decision rule induced by $\cup_{i=1}^{m}(F_i, E_i)$, where and $e_i \in C$ and $G(e_j) \supseteq H(e_i)$ and $e_j \in B$.

Example 12. Let us reconsider the soft set given in Example 4. $(F_1, E_1) \tilde{\cup} (F_2, E_2)$ is a reduct of $(\tilde{\cup}_{i=1}^{3}(F_i, E_i), (F_4, E_4), U)$. The parameter of $(F_1, E_1) \wedge (F_2, E_2)$ are

$e_1 =$ high and good,
$e_2 =$ med and good,
$e_3 =$ low and avg.,
$e_4 =$ med and avg.,
$e_5 =$ high and avg.

The induced rules are:

(1) if high and good then profit (0, 1/4),
(2) if med. and good then profit (3/4, 2/4),
(3) if low and avg. then loss (0, 1/2),
(4) if med. and avg. then loss(1/2, 1),
(5) if high and avg. then profit (1).

4. Application of exclusive disjunctive soft sets

In Ref. [26], Guan and Wang proposed set-valued information system and defined three kinds of relative reducts for set-valued information systems and used them to evaluate the significance of attributes. In their study, they believed incomplete information systems can be viewed as exclusive-disjunctively interpreted set-valued information systems. Moreover, they gave an example of set-valued information system and calculated the relative reducts with their method. In the following, we will use their example to calculate reducts with exclusive disjunctive soft sets.

A set-valued decision table is given in Table 4.1 [26]. In this paper, we take it for example to find the reducts with method based on exclusive disjunctive soft sets. Note that, the fact that the element with more than one value respect to an attribute is exclusive-disjunctively interpreted, which is different from [26]. For example, x_2 have two values {0, 1} when attribute is

Table 4.1
A set-valued decision table (see [26]).

	c_1	c_2	c_3	c_4	c_5	d
x_1	1	0, 1	0	1, 2	2	3
x_2	0, 1	2	1, 2	0	0	1
x_3	0	1, 2	1	0, 1	0	1
x_4	0	1	1	1	0, 2	2
x_5	2	1	0, 1	0	1	2
x_6	0, 2	1	0, 1	0	1	2
x_7	1	0, 2	0, 1	1	2	3
x_8	0	2	1	0	0, 1	1
x_9	1	0, 1	0, 2	1	2	3
x_{10}	1	1	2	0, 1	2	2

Table 4.2
Tabular form of soft set.

	E_1			E_2			E_3			E_4			E_5			B		
	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	1	2	3
x_1		1		1	1		1				1	1			1			1
x_2	1	1				1		1	1	1			1			1		
x_3	1				1	1		1		1	1		1			1		
x_4	1				1			1			1		1		1		1	
x_5			1		1		1	1		1				1			1	
x_6	1		1		1		1	1		1				1			1	
x_7		1		1		1	1	1			1				1			1
x_8	1					1		1		1			1	1		1		
x_9		1		1	1		1		1		1				1			1
x_{10}		1			1				1	1	1				1		1	

Note that, values of the empty cells in this table are '0'.

c_1. It is exclusive-disjunctively interpreted as: when attribute is c_1, the value of x_2 can be 0 or 1, but cannot be both. Therefore, we can use exclusive disjunctive soft sets to obtain the reducts by the algorithm below.

Algorithm

Step 1. Construct exclusive disjunctive soft decision system $((F, E), (G, B), U)$ over universe U for each attribute of set-valued information system, denoted by (F_i, E_i), and bijective soft set for decision attribute d, denoted by (G, B).

Step 2. Calculate each dependency between $\wedge(F_j, E_j)$ and (G, B), where $0 < j \leq n$ and n is the cardinal of condition attribute.

Step 3. Find reduct exclusive disjunctive soft set with respect to $((F, E), (G, B), U)$ by Definition 3.9.

Step 4. Find the core exclusive disjunctive soft set by Definition 3.10.

Let us consider the set-valued decision table in Table 4.1. It can be represented as a soft set in a tabular form in Table 4.2 firstly.

In step 2, we will calculate each dependency between $\wedge(F_j, E_j)$ and (G, B). Here, we will take the computational process of $\gamma((F_1, E_1) \wedge (F_2, E_2), (G, B))$ for example to help better understand how it works.

From Definition 2.6 we can obtain $(F_1, E_1) \wedge (F_2, E_2)$ and the tabular form of $(F_1, E_1) \wedge (F_2, E_2)$ is given in Table 4.3. Suppose that $(F_1, E_1) \wedge (F_2, E_2) = (H, C)$. And we can write:

$$G(1) = \{x_2, x_3, x_8\}, \qquad G(2) = \{x_4, x_5, x_6, x_{10}\}, \qquad G(3) = \{x_1, x_7, x_9\}.$$

From Definition 3.4, we can label entities, contain no uncertain characteristic function, with '*' such as x_4, x_5, x_{10}. And then we can underline entities when each column contains no '*' entities, because when $\underset{\sim}{F}(e)$ is a null set for parameter e (a column of table) any uncertain element can be classified into a $G(c)$, $(c \in B)$. Thus, the entities in columns 1, 3 and 4 are underlined. Next, consider columns 2, 5 and 6. We can write

$$H(1 \wedge 1) = \{x_1/\xi_{x_1}(1 \wedge 1), x_9/\xi_{x_9}(1 \wedge 1), x_{10}\}$$
$$H(0 \wedge 1) = \{x_3/\xi_{x_3}(0 \wedge 1), x_4, x_6/\xi_{x_6}(0 \wedge 1)\}$$
$$H(2 \wedge 1) = \{x_5, x_6/\xi_{x_6}(2 \wedge 1)\}.$$

From Definition 3.4, we can underline the rest columns when each entity is in the result of restricted and operation (see Table 4.3).

Next, we can calculate the dependency by the following algorithm:

Table 4.3
The tabular form of $(F_1, E_1) \wedge (F_2, E_2)$.

	$1 \wedge 0$	$1 \wedge 1$	$0 \wedge 2$	$1 \wedge 2$	$0 \wedge 1$	$2 \wedge 1$
x_1	$\underline{1}$	1				
x_2			$\underline{1}$	$\underline{1}$		
x_3			$\underline{1}$		1	
x_4					$\underline{1^*}$	
x_5						$\underline{1^*}$
x_6					$\underline{1}$	$\underline{1}$
x_7	$\underline{1}$			$\underline{1}$		
x_8			$\underline{1}$			
x_9	$\underline{1}$	1				
x_{10}		$\underline{1^*}$				

Note that, values of the empty cells in this table are ‘0’.

$d = 0$;
$n = 0$.
For each row in result table
  if every ‘1’ entities is l underlined then
    $d = d + 1$;
  else
    $n = n +$ the number of entities without underlined.
  Return $\gamma((H, C), (G, B)) = \frac{d+\chi^{(n)}}{|U|}$.

Thus, we can obtain that $\gamma((F_1, E_1) \wedge (F_2, E_2), (G, B)) = \frac{\chi^{(3)}+7}{10} = \{1, 9/10, 8/10, 7/10\}$.
Some dependencies with bigger value are list:

$$\gamma((F_1, E_1) \wedge (F_2, E_2), (G, B)) = \frac{\chi^{(3)} + 7}{10} = \{1, 9/10, 8/10, 7/10\}$$
$$\gamma((F_2, E_2) \wedge (F_5, E_5), (G, B)) = \frac{\chi^{(2)} + 8}{10} = \{1, 9/10, 8/10\}$$
$$\gamma((F_2, E_2) \wedge (F_3, E_3) \wedge (F_5, E_5), (G, B)) = 1$$
$$\gamma((F_1, E_1) \wedge (F_3, E_3) \wedge (F_4, E_4) \wedge (F_5, E_5), (G, B)) = 1.$$

Note that, $\chi^{(n)}$ denotes that there are n unknown characteristic functions exist in dependency formula and we do not give those exclusive disjunctive soft sets with smaller dependency. The results of reducts show that $(F_2, E_2) \tilde{\cup} (F_3, E_3) \tilde{\cup} (F_5, E_5)$ is better than the other ones. It contains 3 exclusive disjunctive soft sets and its dependency is 1, while $(F_1, E_1) \tilde{\cup} (F_3, E_3) \tilde{\cup} (F_4, E_4) \tilde{\cup} (F_5, E_5)$ contains 4 exclusive disjunctive soft sets despite it has the same dependency as the former.

In step 3, we can obtain the reduct of $((F, E), (G, B), U)$ are:

$$(F_1, E_1) \tilde{\cup} (F_2, E_2)$$
$$(F_2, E_2) \tilde{\cup} (F_5, E_5)$$
$$(F_2, E_2) \tilde{\cup} (F_3, E_3) \tilde{\cup} (F_5, E_5)$$
$$(F_1, E_1) \tilde{\cup} (F_3, E_3) \tilde{\cup} (F_4, E_4) \tilde{\cup} (F_5, E_5).$$

In this step 4, we can obtain that there is no core of $((F, E), (G, B), U)$.

Thus, $(F_2, E_2) \tilde{\cup} (F_3, E_3) \tilde{\cup} (F_5, E_5)$ is a better reduct of $((F, E), (G, B), U)$, which is corresponding with attributes c2, c3, c5 in set-valued decision table (see Table 4.1).

5. Conclusion

The paper proposes a new subtype of soft set, exclusive disjunctive soft sets, and studies some operations of it. It is an extended notion of bijective soft set. This paper also proposes operations, such as, restricted/relaxed AND operation on an exclusive disjunctive soft set and a subset of universe, dependency between exclusive disjunctive soft sets and bijective soft sets, exclusive disjunctive soft decision systems, reduction of exclusive disjunctive soft decision systems, core of exclusive disjunctive soft decision systems, decision rules induced from exclusive disjunctive decision soft decision systems. Moreover, this paper gives an application of exclusive disjunctive soft sets, which shows that it can be applied to attribute reduction of incomplete information system. Further study is very extensive. Potential studies could be focusing on how to apply it to develop algorithm on predicting unknown value of incomplete information system, extending the operations of exclusive disjunctive soft sets to fuzzy soft sets etc., as well as applying it to solve decision-making problems.

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